

## The $B \rightarrow \pi K$ Puzzle: 2009 Update

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### Abstract

The measurements of  $B \rightarrow \pi K$  decays have been in disagreement with the predictions of the Standard Model (SM) for some time. In this paper, we perform an update of this puzzle using the latest (2008) data. We find that the situation has become far less clear. A fit to the  $B \rightarrow \pi K$  data alone suggests the presence of new physics (NP). Indeed, if one adds a constraint on the weak phase  $\gamma$  coming from independent measurements – the SM fit – one finds that the fit is poor. On the other hand, it is not terrible. If one is willing to accept some deficiencies in the fit, it can be argued that the SM can explain the  $B \rightarrow \pi K$  data. If one assumes NP, it is found to be present only in the electroweak penguin amplitude, as before. However, the fit is fair at best, and the improvement over the SM is not particularly strong. All and all, while the  $B \rightarrow \pi K$  puzzle has not disappeared, it has become weaker.

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The four  $B \rightarrow \pi K$  decays –  $B^+ \rightarrow \pi^+ K^0$  (designated as +0 below),  $B^+ \rightarrow \pi^0 K^+$  (0+),  $B_d^0 \rightarrow \pi^- K^+$  (–+) and  $B_d^0 \rightarrow \pi^0 K^0$  (00) – have evoked a great deal of interest in recent years. There are nine measurements of these processes that can be made: the four branching ratios, the four direct CP asymmetries  $A_{CP}$ , and the mixing-induced CP asymmetry  $S_{CP}$  in  $B_d^0 \rightarrow \pi^0 K^0$ . There has been a continual disagreement between this set of measurements and the predictions of the standard model (SM); this discrepancy has been dubbed the “ $B \rightarrow \pi K$  puzzle” [1].

The four  $B \rightarrow \pi K$  amplitudes, which obey a quadrilateral isospin relation, can be written within the diagrammatic approach [2]. Here, the amplitudes are expressed in terms of six diagrams<sup>4</sup>: the color-favored and color-suppressed tree amplitudes  $T'$  and  $C'$ , the gluonic penguin amplitudes  $P'_{tc}$  and  $P'_{uc}$ , and the color-favored and color-suppressed electroweak penguin amplitudes  $P'_{EW}$  and  $P'^C_{EW}$ . (The primes on the amplitudes indicate  $\bar{b} \rightarrow \bar{s}$  transitions.) The amplitudes are given by

$$\begin{aligned} A^{+0} &= -P'_{tc} + P'_{uc} e^{i\gamma} - \frac{1}{3} P'^C_{EW} , \\ \sqrt{2} A^{0+} &= -T' e^{i\gamma} - C' e^{i\gamma} + P'_{tc} - P'_{uc} e^{i\gamma} - P'_{EW} - \frac{2}{3} P'^C_{EW} , \\ A^{-+} &= -T' e^{i\gamma} + P'_{tc} - P'_{uc} e^{i\gamma} - \frac{2}{3} P'^C_{EW} , \\ \sqrt{2} A^{00} &= -C' e^{i\gamma} - P'_{tc} + P'_{uc} e^{i\gamma} - P'_{EW} - \frac{1}{3} P'^C_{EW} . \end{aligned} \quad (1)$$

We have explicitly written the weak-phase dependence (including the minus sign from  $V_{tb}^* V_{ts}$  [in  $P'_{tc}$ ]), while the diagrams contain strong phases. The amplitudes for the CP-conjugate processes can be obtained from the above by changing the sign of the weak phase ( $\gamma$ ).

Within the SM, to a good approximation, the diagrams  $P'_{EW}$  and  $P'^C_{EW}$  can be related to  $T'$  and  $C'$  using flavor SU(3) symmetry<sup>5</sup> [4]:

$$\begin{aligned} P'_{EW} &= \frac{3}{4} \frac{c_9 + c_{10}}{c_1 + c_2} R(T' + C') + \frac{3}{4} \frac{c_9 - c_{10}}{c_1 - c_2} R(T' - C') , \\ P'^C_{EW} &= \frac{3}{4} \frac{c_9 + c_{10}}{c_1 + c_2} R(T' + C') - \frac{3}{4} \frac{c_9 - c_{10}}{c_1 - c_2} R(T' - C') . \end{aligned} \quad (2)$$

Here, the  $c_i$  are Wilson coefficients [5] and  $R \equiv |(V_{tb}^* V_{ts}) / (V_{ub}^* V_{us})|$ . In our fits we take  $R = 48.9 \pm 1.6$  [6].

In Ref. [2], the relative sizes of the  $B \rightarrow \pi K$  diagrams were roughly estimated as

$$1 : |P'_{tc}| , \quad \mathcal{O}(\bar{\lambda}) : |T'|, |P'_{EW}| , \quad \mathcal{O}(\bar{\lambda}^2) : |C'|, |P'_{uc}|, |P'^C_{EW}| , \quad (3)$$

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<sup>4</sup>Note that we have neglected the annihilation diagram  $A'$ , which is expected to be very small in the SM. In any case, its inclusion does not change anything, since the diagrams can be redefined so that the  $B \rightarrow \pi K$  amplitudes are still a function of six diagrams [3].

<sup>5</sup>Note that  $P'_{EW}$  and  $P'^C_{EW}$  are not written with a minus sign in Eq. (1), despite containing the factor  $V_{tb}^* V_{ts}$ . This sign is included in the relations of these diagrams to  $T'$  and  $C'$  [Eq. (2)].

where  $\bar{\lambda} \sim 0.2$ . These estimates have since been modified slightly. First, in realistic models of QCD,  $|C'|$  is allowed to take somewhat larger values:  $|C'/T'| \lesssim 0.6$  [7]. Second, it has been argued that  $|P'_{uc}|$  is actually a bit smaller,  $\mathcal{O}(\bar{\lambda}^3)$  [3].

Before 2006, the  $B \rightarrow \pi K$  puzzle could be seen by comparing two quantities which are expected to be approximately equal in the SM. However, with the data in 2006, it was no longer possible to see such an effect. It was therefore necessary to perform a full fit to the data, and this was done in Ref. [8] (and before that, in Ref. [9]). There it was found that a good fit could be obtained, but at a serious cost. If  $P'_{uc}$  is excluded from the fit, then  $|C'/T'| = 1.6 \pm 0.3$  was required. This is much larger than the allowed value given above. If  $P'_{uc}$  is included in the fit, a smaller value of  $|C'/T'| = 0.8 \pm 0.1$  was obtained. However,  $|P'_{uc}/T'| = 1.7 \pm 0.6$  was found, which is much larger than the expected value above. In addition, one found  $\gamma = (30 \pm 7)^\circ$ , which is inconsistent with independent measurements. In either case, there was a clear indication that a 3-4 $\sigma$  discrepancy between the  $B \rightarrow \pi K$  data and the SM was present in 2006.

The 2008 data are shown in Table 1. Compared to 2006, the only measurements which have changed significantly are  $A_{CP}$  and  $S_{CP}$  in  $B_d^0 \rightarrow \pi^0 K^0$ . However, as we will see, these changes have important consequences.

Mode	$BR[10^{-6}]$	$A_{CP}$	$S_{CP}$
$B^+ \rightarrow \pi^+ K^0$	$23.1 \pm 1.0$	$0.009 \pm 0.025$	
$B^+ \rightarrow \pi^0 K^+$	$12.9 \pm 0.6$	$0.050 \pm 0.025$	
$B_d^0 \rightarrow \pi^- K^+$	$19.4 \pm 0.6$	$-0.098^{+0.012}_{-0.011}$	
$B_d^0 \rightarrow \pi^0 K^0$	$9.8 \pm 0.6$	$-0.01 \pm 0.10$	$0.57 \pm 0.17$

Table 1: Branching ratios, direct CP asymmetries  $A_{CP}$ , and mixing-induced CP asymmetry  $S_{CP}$  (if applicable) for the four  $B \rightarrow \pi K$  decay modes. The data are taken from Refs. [10] and [11].

In this paper, we explore the current status of the  $B \rightarrow \pi K$  puzzle by performing fits to the 2008 data. The  $B \rightarrow \pi K$  amplitudes are written in terms of four independent diagrams and the phase  $\gamma$ . In addition, in  $S_{CP}$  the phase of  $B_d^0$ - $\bar{B}_d^0$  mixing ( $\beta$ ) appears. Our fits therefore involve 9 theoretical parameters: the magnitudes of  $P'_{tc}$ ,  $T'$ ,  $C'$ ,  $P'_{uc}$ , three relative strong phases, and the weak phases  $\beta$  and  $\gamma$ . Note: for  $\beta$ , we add the additional constraint of  $\beta = (21.66^{+0.95}_{-0.87})^\circ$  [6], which is obtained mainly from the measurement of CP violation in  $B_d^0(t) \rightarrow J/\Psi K_S$  and other  $\bar{b} \rightarrow \bar{c} c \bar{s}$  decays. Also, whenever experimental data show asymmetrical errors, we take the larger one as the 1 $\sigma$  standard deviation in the fits.

We first fit to only the  $B \rightarrow \pi K$  data of Table 1. Before presenting the results of this fit, we note the following. In general, several solutions will be found when a fit is done, many of which will indicate new physics (NP). How do we know that one of these is not the correct solution? We can never be absolutely sure, but in order to make progress, we have to make some plausible assumptions. In particular,

we assume that the NP, if present, is not enormous. If it were, it probably would already have been seen elsewhere. In the present case, there is one solution which has a better value of  $\chi^2_{min}/d.o.f.$  than the one eventually chosen. It is shown in Table 2. In the SM,  $|P'_{uc}/P'_{tc}|$  is estimated to be in the range  $\bar{\lambda}^3\text{--}\bar{\lambda}^2$  ( $\bar{\lambda} \sim 0.2$ ). However, in the solution of Table 2, it is found that  $|P'_{uc}| > |P'_{tc}|$ . In order for this to occur, NP must be present, and it must be extremely large. This is counter to our assumption, and we therefore discard this solution on the basis that it cannot represent the true situation.

$\chi^2_{min}/d.o.f.$	$ P'_{tc} $	$ T' $	$ C' $	$ P'_{uc} $
0.02/1	$25.1 \pm 3.6$	$11.6 \pm 2.1$	$9.1 \pm 2.4$	$42.9 \pm 3.2$

Table 2: Discarded solution of the fit to  $P'_{tc}$ ,  $T'$ ,  $C'$ ,  $P'_{uc}$ ,  $\beta$  and  $\gamma$  in the SM. The fit includes the constraint  $\beta = (21.66^{+0.95}_{-0.87})^\circ$ . The amplitude is in units of eV.

The solution which is retained is shown in Table 3. Note: throughout this paper, we adopt the convention that the strong phase of  $P'_{tc}$  is zero. Now, the underlying diagram  $P'_{tc}$  does have a strong phase, coming from rescattering of the  $\bar{b} \rightarrow \bar{c}c\bar{s}$  tree diagram<sup>6</sup>. So we have shifted the strong phases of all diagrams by this quantity. Thus, when we write  $\delta_{T'}$ , it really corresponds to the strong phase of the underlying  $T'$  diagram minus  $\delta_{P'_{tc}}$ , and similarly for  $\delta_{C'}$  and  $\delta_{P'_{uc}}$  (and  $\delta_{NP}$  later in the paper).

We find that  $\chi^2_{min}/d.o.f. = 0.52/1$  (47%). (The number in parentheses indicates the quality of the fit, and depends on  $\chi^2_{min}$  and  $d.o.f.$  individually. 50% or more is a good fit; fits which are substantially less than 50% are poorer.) Here the quality of the fit is fair. However, there is clearly a possible hint of NP. The most important point is that the value of  $\gamma$  extracted is very far from that obtained from independent measurements,  $\gamma \sim 67^\circ$ . This leads to a discrepancy of  $3.5\sigma$ . (It is also true that the central value of  $|P'_{uc}|$  is very large. But the error is also large, so this is not as much of a problem.)

$\chi^2_{min}/d.o.f.$	$ P'_{tc} $	$ T' $	$ C' $	$ P'_{uc} $
0.52/1	$67.7 \pm 11.8$	$19.6 \pm 6.9$	$14.9 \pm 6.6$	$20.5 \pm 13.3$
$\delta_{T'}$	$\delta_{C'}$	$\delta_{P'_{uc}}$	$\beta$	$\gamma$
$(6.0 \pm 4.0)^\circ$	$(-11.7 \pm 6.8)^\circ$	$(-0.7 \pm 2.3)^\circ$	$(21.66 \pm 0.95)^\circ$	$(35.3 \pm 7.1)^\circ$

Table 3: Results of the fit to  $P'_{tc}$ ,  $T'$ ,  $C'$ ,  $P'_{uc}$ ,  $\beta$  and  $\gamma$  in the SM. The fit includes the constraint  $\beta = (21.66^{+0.95}_{-0.87})^\circ$ . The amplitude is in units of eV.

<sup>6</sup>This is theory input. Experimentally, one can only measure differences of strong phases. However, there is a theoretical framework describing the generation of individual strong phases. This framework of rescattering is used in numerous papers, and is included in all theories of QCD. We use it throughout this paper.

Since the quality of fit is only fair, it is difficult to conclude definitively that NP is indicated. We therefore perform a second fit, in which an additional constraint on  $\gamma$  from independent measurements has been imposed<sup>7</sup>:  $\gamma = (66.8^{+5.4}_{-3.8})^\circ$  [6]. This is the “SM fit.” This fit may not be good, but the question is: how bad is it? Could the SM still be considered to explain the  $B \rightarrow \pi K$  data?

There is one discarded solution; that which is kept is shown in Table 4. We find that  $\chi^2_{min}/d.o.f. = 3.2/2$  (20%). In addition to the poor quality of fit, the ratio  $|T'/P'_{tc}|$  is on the small side. Neither of these deficiencies definitively excludes the SM as the explanation of the  $B \rightarrow \pi K$  data. However, they do negatively impact the SM fit. Nevertheless, one might consider that the  $B \rightarrow \pi K$  measurements are not actually at odds with the SM. One therefore sees that the  $B \rightarrow \pi K$  puzzle is now in an uncertain situation – it is not clear if NP is indicated, or if the SM is sufficient.

$\chi^2_{min}/d.o.f.$	$ P'_{tc} $	$ T' $	$ C' $	$ P'_{uc} $
3.2/2	$50.5 \pm 1.8$	$5.9 \pm 1.8$	$3.4 \pm 1.0$	$2.3 \pm 4.9$
$\delta_{T'}$	$\delta_{C'}$	$\delta_{P'_{uc}}$	$\beta$	$\gamma$
$(25.5 \pm 11.2)^\circ$	$(252.0 \pm 36.1)^\circ$	$(-9.9 \pm 27.8)^\circ$	$(21.65 \pm 0.95)^\circ$	$(66.5 \pm 5.5)^\circ$

Table 4: Results of the fit to  $P'_{tc}$ ,  $T'$ ,  $C'$ ,  $P'_{uc}$ ,  $\beta$  and  $\gamma$  in the SM. The fit includes the constraints  $\beta = (21.66^{+0.95}_{-0.87})^\circ$  and  $\gamma = (66.8^{+5.4}_{-3.8})^\circ$ . The amplitude is in units of eV.

Recent analyses have examined new contributions to  $\bar{b} \rightarrow \bar{s}$  transitions [12]. When looking at  $B \rightarrow \pi K$  decays, they have focused on the difference between  $S_{CP}(\pi^0 K^0)$  and the measured indirect CP asymmetry in  $B_d^0 \rightarrow J/\Psi K_s$ . However, we find that there is little difficulty in reproducing the measured value of  $S_{CP}(\pi^0 K^0)$  in the fits. (Also, the fact that there is a greater than  $5\sigma$  difference between  $A_{CP}(\pi^0 K^+)$  and  $A_{CP}(\pi^- K^+)$  [8] is not a problem once the smaller diagrams  $P'_{EW}$ ,  $C'$  and  $P'^C_{EW}$  are taken into account [13].) Rather, it is the direct CP asymmetry in  $B_d^0 \rightarrow \pi^0 K^0$  which causes the most problems. This is illustrated in Table 5, in which the predictions for each of the observables are given for the fits of Table 3 (Fit 1) and Table 4 (Fit 2), along with the “pull” from the data. (The pull is defined as (data central value – theory prediction) / (data error).) We see that  $A_{CP}(\pi^0 K^0)$  has the largest pull in both fits (and that  $S_{CP}(\pi^0 K^0)$  has a small pull).

There is a slight possible complication, related to the  $A_{CP}(\pi^0 K^0)$  measurement by the BaBar Collaboration ( $-0.13 \pm 0.13$ ) and the Belle Collaboration ( $0.14 \pm 0.14$ ) – the central values do not quite agree with each other. (The average is nearly zero, which is used in our previous fits.) However, both of our predictions in Table 5 favor the BaBar measurement. We therefore also consider the above-mentioned two fits

<sup>7</sup>There are no  $\bar{b} \rightarrow \bar{s}$  measurements entering this value of  $\gamma$ , so there is no inconsistency in using it in the search for  $\bar{b} \rightarrow \bar{s}$  NP in the  $B \rightarrow \pi K$  puzzle.

Obs.	Fit 1	Fit 2
$BR(\pi^+ K^0)$	23.1 (+0.02)	23.7 (−0.57)
$A_{CP}(\pi^+ K^0)$	0.014 (−0.21)	0.016 (−0.29)
$BR(\pi^0 K^+)$	12.9 (−0.03)	12.5 (+0.72)
$A_{CP}(\pi^0 K^+)$	0.05 (+0.15)	0.04 (+0.27)
$BR(\pi^- K^+)$	19.4 (+0.05)	19.7 (−0.46)
$A_{CP}(\pi^- K^+)$	−0.098 (−0.04)	−0.097 (−0.12)
$BR(\pi^0 K^0)$	9.8 (−0.07)	9.3 (+0.88)
$A_{CP}(\pi^0 K^0)$	−0.08 (+0.66)	−0.12 (+1.10)
$S_{CP}(\pi^0 K^0)$	0.58 (−0.03)	0.58 (−0.08)

Table 5: Predictions of the  $B \rightarrow \pi K$  decay observables based upon the best-fitted results in Table 3 (Fit 1) and Table 4 (Fit 2). Branching ratios are given in units of  $10^{-6}$ . Numbers in parentheses are the corresponding pulls.

using the BaBar value for  $A_{CP}(\pi^0 K^0)$ . The results are shown in Table 6. Indeed, we observe an improved quality of fit (from 47% and 20% to 76% and 43%, respectively). On the other hand, despite this improvement, there are similar puzzling features as found before. For example, Fit 1' still gives a value of  $\gamma$  about  $3.5\sigma$  below other determinations of this quantity. And even though the ratio  $|T'/P'_{tc}|$  in Fit 2' is found to be larger than that in Fit 2, a new problem arises in that  $|C'/T'| = 1.6 \pm 0.6$ . We note in passing (without presenting all the details) that if we take instead the Belle measurement for  $A_{CP}(\pi^0 K^0)$  in the two fits, the fit quality drops to 18% and 7%, respectively. The results corresponding to Fit 1 have the problems that  $\gamma = 96.4 \pm 12.4$ , about  $2.2\sigma$  above the other measurements, and  $|P'_{uc}| = 31.9 \pm 5.4$  is large. The problem in the results corresponding to Fit 2 is still the small  $|T'/P'_{tc}|$  ratio.

Assuming that new physics is present in  $B \rightarrow \pi K$ , it is now important to know what type of NP it is. We follow the approach developed in Ref. [14]. All NP operators in  $B \rightarrow \pi K$  decays take the form  $\mathcal{O}_{NP}^{ij,q} \sim \bar{s}\Gamma_i b \bar{q}\Gamma_j q$  ( $q = u, d$ ), where  $\Gamma_{i,j}$  represent Lorentz structures, and color indices are suppressed. These operators contribute through the matrix elements  $\langle \pi K | \mathcal{O}_{NP}^{ij,q} | B \rangle$ . In general, each matrix element has its own NP weak and strong phases.

In Ref. [14], it is argued that all NP strong phases are negligible. The reason is that the strong phase of the SM diagram  $P'_{tc}$  is generated by rescattering of the  $\bar{b} \rightarrow \bar{c}c\bar{s}$  tree diagram, which is about 100 times as big. On the other hand, the NP strong phase can only be generated by rescattering of the NP diagram itself, i.e. self-rescattering. It is therefore negligible compared to the strong phase of  $P'_{tc}$ . In this case, one can combine all NP matrix elements into a single NP amplitude, with a single weak phase:

$$\sum \langle \pi K | \mathcal{O}_{NP}^{ij,q} | B \rangle = \mathcal{A}^q e^{i\Phi_q} . \quad (4)$$

$\chi^2_{min}/d.o.f.$	$ P'_{tc} $	$ T' $	$ C' $	$ P'_{uc} $
Fit 1': 0.095/1	$66.9 \pm 11.9$	$18.8 \pm 7.3$	$14.1 \pm 7.0$	$19.9 \pm 13.3$
$\delta_{T'}$	$\delta_{C'}$	$\delta_{P'_{uc}}$	$\beta$	$\gamma$
$(5.4 \pm 4.3)^\circ$	$(-13.6 \pm 8.4)^\circ$	$(0.0 \pm 2.4)^\circ$	$(21.66 \pm 0.95)^\circ$	$(35.9 \pm 7.7)^\circ$
$\chi^2_{min}/d.o.f.$	$ P'_{tc} $	$ T' $	$ C' $	$ P'_{uc} $
Fit 2': 1.7/2	$41.5 \pm 2.4$	$8.5 \pm 2.5$	$13.7 \pm 2.6$	$10.7 \pm 3.9$
$\delta_{T'}$	$\delta_{C'}$	$\delta_{P'_{uc}}$	$\beta$	$\gamma$
$(139.2 \pm 12.8)^\circ$	$(212.2 \pm 7.2)^\circ$	$(184.0 \pm 4.3)^\circ$	$(21.59 \pm 0.95)^\circ$	$(64.6 \pm 4.9)^\circ$

Table 6: Results of the fits to  $P'_{tc}$ ,  $T'$ ,  $C'$ ,  $P'_{uc}$ ,  $\beta$  and  $\gamma$  in the SM. The averaged experimental data given in Table 1 is used, except that only the BaBar measurement of  $A_{CP}(\pi^0 K^0) = -0.13 \pm 0.13$  is taken. The fits include the constraints  $\beta = (21.66^{+0.95}_{-0.87})^\circ$ . For  $\gamma$ , we impose no constraint (Fit 1'), or  $\gamma = (66.8^{+5.4}_{-3.8})^\circ$  (Fit 2'). The amplitude is in units of eV.

There are two classes of NP amplitudes, differing only in their color structure:  $\bar{s}_\alpha \Gamma_i b_\alpha \bar{q}_\beta \Gamma_j q_\beta$  and  $\bar{s}_\alpha \Gamma_i b_\beta \bar{q}_\beta \Gamma_j q_\alpha$  ( $q = u, d$ ). They are denoted  $\mathcal{A}'^q e^{i\Phi'_q}$  and  $\mathcal{A}'^{C,q} e^{i\Phi'^C_q}$ , respectively [15]. Here,  $\Phi'_q$  and  $\Phi'^C_q$  are the NP weak phases; the strong phases are zero. Each of these contributes differently to the various  $B \rightarrow \pi K$  decays. In general,  $\mathcal{A}'^q \neq \mathcal{A}'^{C,q}$  and  $\Phi'_q \neq \Phi'^C_q$ . Note: despite the “color-suppressed” index  $C$ , the matrix elements  $\mathcal{A}'^{C,q} e^{i\Phi'^C_q}$  are not necessarily smaller than  $\mathcal{A}'^q e^{i\Phi'_q}$ .

There are three NP matrix elements which contribute to the  $B \rightarrow \pi K$  amplitudes:  $\mathcal{A}'^{comb} e^{i\Phi'} \equiv -\mathcal{A}'^{u} e^{i\Phi'_u} + \mathcal{A}'^{d} e^{i\Phi'_d}$ ,  $\mathcal{A}'^{C,u} e^{i\Phi'^C_u}$ , and  $\mathcal{A}'^{C,d} e^{i\Phi'^C_d}$  [15]. The first operator corresponds to including NP only in the color-favored electroweak penguin amplitude:  $\mathcal{A}'^{comb} e^{i\Phi'} \equiv -P'_{EW,NP} e^{i\Phi'_{EW}}$ . Nonzero values of  $\mathcal{A}'^{C,u} e^{i\Phi'^C_u}$  and/or  $\mathcal{A}'^{C,d} e^{i\Phi'^C_d}$  imply the inclusion of NP in both the gluonic and color-suppressed electroweak penguin amplitudes,  $P'_{NP} e^{i\Phi'_P}$  and  $P'^C_{EW,NP} e^{i\Phi'^C_{EW}}$ , respectively [16]:

$$\begin{aligned}
P'_{NP} e^{i\Phi'_P} &\equiv \frac{1}{3} \mathcal{A}'^{C,u} e^{i\Phi'^C_u} + \frac{2}{3} \mathcal{A}'^{C,d} e^{i\Phi'^C_d}, \\
P'^C_{EW,NP} e^{i\Phi'^C_{EW}} &\equiv \mathcal{A}'^{C,u} e^{i\Phi'^C_u} - \mathcal{A}'^{C,d} e^{i\Phi'^C_d}.
\end{aligned} \tag{5}$$

(In Ref. [9], NP only in the gluonic penguin amplitude was referred to as “isospin-conserving NP:”  $\mathcal{A}'^{C,u} e^{i\Phi'^C_u} = \mathcal{A}'^{C,d} e^{i\Phi'^C_d}$ ,  $\mathcal{A}'^{comb} e^{i\Phi'} = 0$ .)

The  $B \rightarrow \pi K$  amplitudes can now be written in terms of the SM amplitudes, along with the NP matrix elements. We neglect only the (small) SM diagram  $P'_{uc}$ :

$$\begin{aligned}
A^{+0} &= -P'_{tc} - \frac{1}{3} P'^C_{EW} + P'_{NP} e^{i\Phi'_P} - \frac{1}{3} P'^C_{EW,NP} e^{i\Phi'^C_{EW}}, \\
\sqrt{2} A^{0+} &= P'_{tc} - T' e^{i\gamma} - P'_{EW} - C' e^{i\gamma} - \frac{2}{3} P'^C_{EW} \\
&\quad - P'_{EW,NP} e^{i\Phi'_{EW}} - P'_{NP} e^{i\Phi'_P} - \frac{2}{3} P'^C_{EW,NP} e^{i\Phi'^C_{EW}},
\end{aligned}$$

$$\begin{aligned}
A^{-+} &= P'_{tc} - T' e^{i\gamma} - \frac{2}{3} P'^C_{EW} - P'_{NP} e^{i\Phi'_P} - \frac{2}{3} P'^C_{EW,NP} e^{i\Phi'^C_{EW}} , \\
\sqrt{2} A^{00} &= -P'_{tc} - P'_{EW} - C' e^{i\gamma} - \frac{1}{3} P'^C_{EW} \\
&\quad - P'_{EW,NP} e^{i\Phi'_{EW}} + P'_{NP} e^{i\Phi'_P} - \frac{1}{3} P'^C_{EW,NP} e^{i\Phi'^C_{EW}} . \tag{6}
\end{aligned}$$

Even if the value of  $\gamma$  is taken from independent measurements, there are too many theoretical parameters to perform a fit containing all three NP operators. It is therefore necessary to make some theoretical assumptions. As in Ref. [9], we assume that a single NP amplitude dominates, and consider  $P'_{NP} e^{i\Phi'_P}$ ,  $P'_{EW,NP} e^{i\Phi'_{EW}}$ , and  $P'^C_{EW,NP} e^{i\Phi'^C_{EW}}$  individually.

However, we have not included all the information at our disposal. The strong phase of the  $T'$  diagram can arise only from self-rescattering. Thus, like the NP amplitudes, this strong phase is expected to be very small. We take this into account by adding the constraint  $\delta_{T'} = \delta_{NP}$ .

The results of the NP fits are given in Table 7. In the first fit,  $P'_{NP} e^{i\Phi'_P}$  is assumed to be nonzero. Several of the entries in the Table are given as NA. This stands for “not applicable,” and means that there are no limits on the corresponding theoretical parameters. This can be understood as follows. This type of NP always appears in the following combination in the  $B \rightarrow \pi K$  amplitudes:  $P'_{tc} - P'_{NP} e^{i\Phi'_P}$ . This contains the 4 quantities  $|P'_{tc}|$ ,  $|P'_{NP}|$ ,  $\delta_{NP}$ <sup>8</sup>, and  $\Phi'_P$ . However, here these are not all independent. The easiest way to see this is to use the convention in which  $e^{i\delta_{P'_{tc}}}$  multiplies  $|P'_{tc}|$ . For  $B$  and  $\bar{B}$  decays, the combinations are:

$$\begin{aligned}
\tilde{P}'_{tc} e^{i\delta_{P'_{tc}}} - \tilde{P}'_{NP} e^{i\Phi'_P} &\equiv z , \\
\tilde{P}'_{tc} e^{i\delta_{P'_{tc}}} - \tilde{P}'_{NP} e^{-i\Phi'_P} &\equiv z' . \tag{7}
\end{aligned}$$

Here, the diagrams are written with a tilde to indicate the different convention, and that the strong phases are given explicitly.  $z$  and  $z'$  are complex numbers; their 4 real and imaginary parts can be written in terms of the 4 theoretical parameters. However, it is clear from the above expressions that  $\text{Re } z = \text{Re } z'$ . The 4 parameters  $|P'_{tc}|$ ,  $|P'_{NP}|$ ,  $\delta_{NP}$  and  $\Phi'_P$  are therefore not independent, and so their allowed ranges cannot be fixed; they are given as NA in Table 7.  $\delta_{C'}$  only appears in  $B \rightarrow \pi K$  observables in tandem with other strong phases. These are NA, so that  $\delta_{C'}$  is as well. Since there are no constraints on the NP parameters, this fit [ $\chi^2_{min}/d.o.f. = 3.6/2$  (17%)] is basically that of the SM. Indeed, this result is quite similar to that given in Table 4 (the small differences are due to the fact that  $P'_{uc}$  is neglected here).

This shows that global fits are insensitive to NP in the gluonic penguin amplitude. If one wishes to investigate the possibility that such NP can account for the  $B \rightarrow \pi K$

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<sup>8</sup>In our convention,  $\delta_{NP} = \delta_{new \text{ phys}} - \delta_{P'_{tc}}$ . However, the new-physics strong phase  $\delta_{new \text{ phys}}$  is negligible, so that  $\delta_{NP} = -\delta_{P'_{tc}}$ .



$\chi^2_{min}/d.o.f.$	$ P'_{tc} $	$ T' $	$ C' $
3.6/2	NA	$5.9 \pm 2.0$	$3.6 \pm 1.0$
$ P'_{NP} $	$\delta_{C'}$	$\delta_{NP}$	$\Phi'_P$
NA	NA	NA	NA
$\chi^2_{min}/d.o.f.$	$ P'_{tc} $	$ T' $	$ C' $
0.4/2	$48.2 \pm 1.3$	$2.6 \pm 0.4$	$16.1 \pm 28.4$
$ P'_{EW,NP} $	$\delta_{C'}$	$\delta_{NP}$	$\Phi'_{EW}$
$20.1 \pm 22.3$	$(254.8 \pm 21.8)^\circ$	$(95.4 \pm 9.6)^\circ$	$(37.6 \pm 51.8)^\circ$
$\chi^2_{min}/d.o.f.$	$ P'_{tc} $	$ T' $	$ C' $
2.5/2	$48.2 \pm 1.3$	$1.9 \pm 1.4$	$9.4 \pm 2.3$
$ P'^C_{EW,NP} $	$\delta_{C'}$	$\delta_{NP}$	$\Phi'^C_{EW}$
$16.5 \pm 15.2$	$(192.4 \pm 12.3)^\circ$	$(97.8 \pm 15.3)^\circ$	$(183.9 \pm 7.8)^\circ$

Table 7: Results of the fits to  $P'_{tc}$ ,  $T'$ ,  $C'$ ,  $\beta$ ,  $\gamma$  and a NP amplitude. The fits include the constraints  $\beta = (21.66^{+0.95}_{-0.87})^\circ$  and  $\gamma = (66.8^{+5.4}_{-3.8})^\circ$ , and in all cases, the best-fit values of  $\beta$  and  $\gamma$  are consistent with these. The constraint  $\delta_{T'} = \delta_{NP}$  is also added. The amplitude is in units of eV. The entry NA (not applicable) is explained in the text.

data, it is necessary to consider specific models and perform a model-dependent calculation. Note: in previous analyses, the NP operators  $\mathcal{A}^{IC,u} e^{i\Phi'^C_u}$  and  $\mathcal{A}^{IC,d} e^{i\Phi'^C_d}$  were considered, and constraints on the theoretical parameters given. From the above, we see that these constraints are due solely to NP in the color-suppressed electroweak penguin.

In the second fit,  $P'_{EW,NP} e^{i\Phi'_{EW}}$  is assumed to be nonzero. In this case, the fit is excellent:  $\chi^2_{min}/d.o.f. = 0.4/2$  (82%). However, the central value of  $|C'/T'|$  is enormous, far outside of the allowed range [Eq. (3)]. Even though the errors are large, this is worrisome. The third fit takes  $P'^C_{EW,NP} e^{i\Phi'^C_{EW}}$  to be nonzero. Here the fit is poor –  $\chi^2_{min}/d.o.f. = 2.5/2$  (28%) – but might still be considered as viable. On the other hand, even here  $|C'/T'|$  is far too large, which poses problems for this scenario.

Because the value of  $|C'/T'|$  is large in two of the fits, we redo the fits with the constraint  $|C'/T'| = 0.5$ . The results are given in Table 8. As before, the fit with  $P'_{NP} e^{i\Phi'_P}$  just reproduces that of the SM. The improved quality of fit –  $\chi^2_{min}/d.o.f. = 3.7/3$  (29%) – simply corresponds to the fact that the d.o.f. has increased from 2 to 3. In the second fit, with  $P'_{EW,NP} e^{i\Phi'_{EW}} \neq 0$ , the quality of fit has decreased markedly:  $\chi^2_{min}/d.o.f. = 3.0/3$  (39%). While this is still better than the SM, the improvement with NP is hardly convincing (and the fit is at best fair). The third fit has a nonzero  $P'^C_{EW,NP} e^{i\Phi'^C_{EW}}$ . It is not particularly good:  $\chi^2_{min}/d.o.f. = 3.8/3$  (28%). In it, the value of the NP parameter is rather small, so that this scenario is also essentially that of the SM.

$\chi^2_{min}/d.o.f.$	$ P'_{tc} $	$ T' $	$ P'_{NP} $
3.7/3	NA	$6.6 \pm 1.1$	NA
$\delta_{C'}$	$\delta_{NP}$	$\Phi'_P$	
NA	NA	NA	
$\chi^2_{min}/d.o.f.$	$ P'_{tc} $	$ T' $	$ P'_{EW,NP} $
3.0/3	$48.0 \pm 0.6$	$2.6 \pm 0.3$	$15.7 \pm 3.6$
$\delta_{C'}$	$\delta_{NP}$	$\Phi'_{EW}$	
$(182.5 \pm 53.1)^\circ$	$(98.4 \pm 4.7)^\circ$	$(-11.6 \pm 5.7)^\circ$	
$\chi^2_{min}/d.o.f.$	$ P'_{tc} $	$ T' $	$ P'^C_{EW,NP} $
3.8/3	$49.8 \pm 0.7$	$6.5 \pm 1.4$	$2.1 \pm 6.2$
$\delta_{C'}$	$\delta_{NP}$	$\Phi'^C_{EW}$	
$(274.7 \pm 59.2)^\circ$	$(15.6 \pm 10.8)^\circ$	$(69.7 \pm 67.4)^\circ$	

Table 8: Results of the fits to  $P'_{tc}$ ,  $T'$ ,  $C'$ ,  $\beta$ ,  $\gamma$  and a NP amplitude. The fits include the constraints  $\beta = (21.66^{+0.95}_{-0.87})^\circ$  and  $\gamma = (66.8^{+5.4}_{-3.8})^\circ$ , and in all cases, the best-fit values of  $\beta$  and  $\gamma$  are consistent with these. The constraints  $\delta_{T'} = \delta_{NP}$  and  $|C'/T'| = 0.5$  are also added. The amplitude is in units of eV. The entry NA (not applicable) is explained in the text.

If there is NP, the  $B \rightarrow \pi K$  data point to the color-favored electroweak penguin amplitude. This is the same situation as in previous analyses. However, whereas in the past, this was a  $3\text{-}4\sigma$  effect, now it is much less clear. Both the SM and NP fits are only fair, and the NP case is an improvement on the SM only by a small amount.

In summary, we have performed an update of the  $B \rightarrow \pi K$  puzzle by performing several fits comparing the  $B \rightarrow \pi K$  measurements as of 2008 with the predictions of the Standard Model (SM). Our first fit involves only the  $B \rightarrow \pi K$  data. We find a possible hint of new physics (NP), since the extracted value of the weak phase  $\gamma$  disagrees with that of independent measurements. However, the fit is only fair. We then constrain  $\gamma$  to satisfy these measurements and perform another fit (the SM fit). We find that the fit is on the poor side. However, it is not so bad that it is excluded. We now have the situation where the  $B \rightarrow \pi K$  data somewhat favor NP over the SM, but not by a huge amount.

We now assume that NP is present. It can appear in the gluonic penguin, the color-favored electroweak penguin, and/or the color-suppressed electroweak penguin. We consider each of these NP scenarios individually. We first show that the global fit is insensitive to NP in the gluonic penguin – with this type of NP, one simply reproduces the result of the SM. If NP is in the color-suppressed electroweak penguin, the best fit has a small contribution of NP. Thus, this is also very much like the SM. The only case in which one finds a large, nonzero NP operator is when it contributes to the color-favored electroweak penguin amplitude. This appears to be

as in previous analyses. The difference is that now this fit is only fair, and it is better than that of the SM by only a small amount.

The conclusion is that, while the  $B \rightarrow \pi K$  puzzle is still present, it is considerably weaker. Neither the SM nor NP gives an excellent fit to the data. And while the  $B \rightarrow \pi K$  measurements do point to NP in the color-favored electroweak penguin, this is not a clear indication, as it was before, and the NP scenario is only a little better than that of the SM.

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